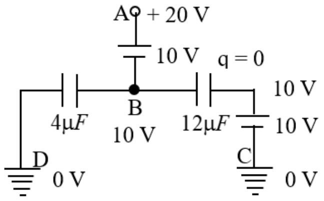
Solutions to JEE MAIN – 3 | JEE 2024

PHYSICS

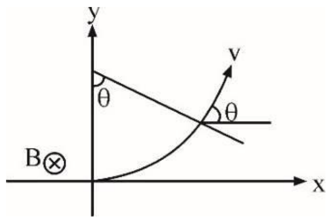
SECTION-1

1.(A) Charge on $12\mu F$ capacitor = 12(10 – 10) = 0

Charge on $4\mu F$ capacitor = 4(10 – 0) = 40



2.(A) $T = \dfrac{2\pi m}{qB} = \dfrac{2\pi}{q/mB} \quad = \dfrac{2\pi}{\pi(2)} = 1\text{sec}$



$\Rightarrow \qquad \theta = \dfrac{2\pi}{6} = \pi/3 = 60°$

$\bar{v} = v(\cos\theta i + \sin\theta j) \; = 10\left[\dfrac{1}{2}\hat{i} + \dfrac{\sqrt{3}}{2}j\right] = 5\hat{i} + 5\sqrt{3}\,j \; m/s$

3.(B) $\dfrac{\frac{3l}{4}}{\frac{3R}{4}} \;\; \dfrac{\frac{l}{4}}{\frac{R}{4}}$

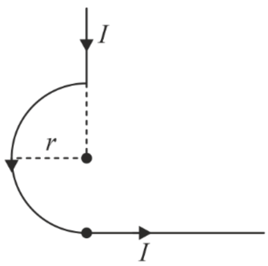
Resistance of wire after stretching will be

$R'_{eq} = R_1 + R'_2$

$= \dfrac{3R}{4} + \dfrac{R}{4}(1.25)^2 = \dfrac{3R}{4} + \dfrac{R}{4} \times \dfrac{25}{16} = \dfrac{73R}{64}$

% change $= \dfrac{R'_{eq} - R_{eq}}{R_{eq}} \times 100 = \dfrac{\frac{73R}{64} - R}{R} \times 100 \; = \dfrac{900}{64} = 14.06$

4.(C) $B_0 = \left|\vec{B}_1 + \vec{B}_2 + \vec{B}_3\right|$

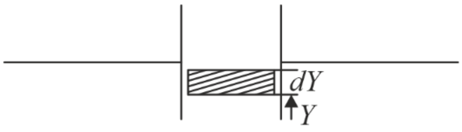


$B_1 = 0 \Rightarrow B_0 = B_2 + B_3$

$\Rightarrow \qquad B_0 = \dfrac{1}{2}\left(\dfrac{\mu_o I}{2r}\right) + \dfrac{1}{2}\left(\dfrac{\mu_o I}{2\pi r}\right)$

$\Rightarrow \qquad B_0 = \dfrac{\mu_o I}{4r} + \dfrac{\mu_o I}{4\pi r}$

5.(C) $dA = adY; \qquad K = K_0(1+\alpha Y)$



Capacitance of element $= \dfrac{K \in_0 adY}{d}$

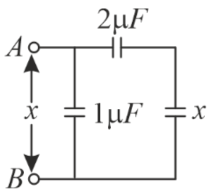
$dC = \dfrac{K_0(1+\alpha Y)\in_0 adY}{d}; \qquad C_{eq} = \int dC$

$C_{eq} = \int \dfrac{K_0(1+\alpha Y)\in_0 adY}{d}; \quad C_{eq} = \dfrac{K_0 \in_0 a}{d}\int_0^a (1+\alpha Y)dY$

$C_{eq} = \dfrac{K_0 \in_0 a}{d}\left(a + \dfrac{\alpha a^2}{2}\right); \qquad C_{eq} = \dfrac{K_0 \in_0 a^2}{d}\left(1+\dfrac{\alpha a}{2}\right)$

6.(B) $\overline{F} = i\left(\overline{\ell}\times\overline{B}\right) = 1\left[\sqrt{2}\left(\hat{i} + j\right)\times\left(3\hat{i} + 4j + k\right)\right] = \left[\sqrt{2}\left(\hat{i} - j + k\right)\right]$

7.(B)



(Let $C_{eq} = x$)

$x = \dfrac{2x}{2+x} + 1$

$x = \dfrac{2x+2+x}{2+x}$

$\Rightarrow x(2+x) = 3x+2$

$\Rightarrow 2x + x^2 = 3x+2 \Rightarrow x^2 - x - 2 = 0$

Use

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a} = \dfrac{1\pm\sqrt{1+8}}{2} = \dfrac{1\pm 3}{2} = 2$ and -1

$x = 2, C_{eq} = 2\mu F$

8.(D) Given, $\vec{B} = B_0 j$

$\vec{F} = I(\vec{l}\times\vec{B})$

$\therefore$ Force on wire 1, $\vec{F_1} = i\left(ak \times B_0 j\right) = -iaB_0\left(\hat{i}\right)$

Force on wire 2, $\vec{F_2} = i\left[\left(-a\hat{i}\right)\times B_0 j\right] = -iaB_0\left(+k\right)$

Force on wire 3, $\vec{F_3} = i\left(a\hat{j}\times B_0 j\right) = \vec{0}$

Force on wire 4, $\vec{F_4} = i\left[a\left(-k\right)\times B_0 j\right] = iaB_0\left(\hat{i}\right)$

i.e., force on wire 4 is iaB_0 in positive X-direction.

9.(C) There are three capacitors in parallel.

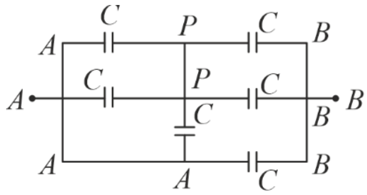
10.(D) $r = \dfrac{mv}{qB} = \dfrac{\sqrt{2mK}}{qB}$ $\quad \therefore \quad r \propto \dfrac{\sqrt{m}}{q}$ [K.E. is some for all particles]

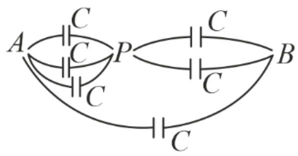
$m_{H_e^{+2}} = 4m_p$; $q_{H_e^{+2}} = 2q_p$; $m_p > m_e$; $q_p = q_e$

$r_p = k\dfrac{\sqrt{m_p}}{q_p}$; $k = \dfrac{\sqrt{2K}}{B}$; $r_e = \dfrac{k\sqrt{m_e}}{q_e}$; $r_{He^{+2}} = \dfrac{k\sqrt{4m_p}}{2q_p} = \dfrac{k\sqrt{m_p}}{q_p}$

$\therefore \quad r_e < r_p = r_{He}$

11.(B)





Get $\quad C_{eq} = \dfrac{11C}{5}$

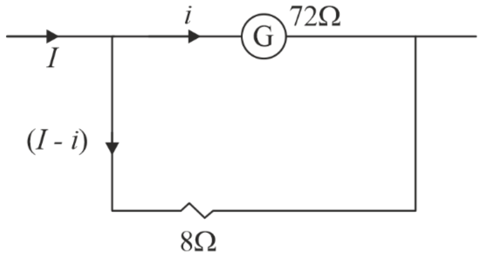
Charge flow $= C_{eq}\varepsilon = \dfrac{11C\varepsilon}{5}$

12.(A) $\dfrac{R_1 + 10}{R_2} = \dfrac{50}{50}$ …(i) [Case-1]

$\dfrac{R_1}{R_2} = \dfrac{40}{60}$ …(ii) [Case-2]

An solving (i) and (ii), we get: $R_1 = 20\Omega$ and $R_2 = 30\Omega$

13.(B)



$72i = 8(I - i) \quad \Rightarrow \quad i = \dfrac{8I}{80} = \dfrac{I}{10}$

Percentage of total current $= \dfrac{\frac{I}{10}}{I} \times 100 = 10\%$

14.(B) Magnitude of torque is given by $|\vec{\tau}| = MB\sin\theta$

Here, $M = NiA = (1)(1.0)(\pi)(0.2)^2 = (0.04\pi)\,\text{A} - \text{m}^2$

and θ = angle between $\vec{M}$ and $\vec{B} = 90^0$

$\therefore \quad |\vec{\tau}| = (0.04\pi)(2)\sin 90^0 = 0.08\pi\,\text{N - m}$

15.(A) $R_{eq} = \dfrac{\dfrac{v^2}{60} \times \dfrac{v^2}{30}}{\dfrac{v^2}{60} + \dfrac{v^2}{30}}$

$R_{eq} = \dfrac{v^2}{90}$

$i = \dfrac{v}{R_{eq}} = \dfrac{v}{v^2/90}$; $i = \dfrac{90}{V} = \dfrac{90}{200} = 0.45A$

16.(B) $B_{net} = \dfrac{\mu_0 l}{4\pi r}[\sin\alpha + \sin\beta] \times 2$

Where $r = \dfrac{a}{2}\cos 45°$

$\alpha = \beta = 45°$

$B_{net} = \dfrac{\mu_0 l}{4\pi \cdot \dfrac{a}{2\sqrt{2}}}\left[\dfrac{1}{\sqrt{2}} \times 2 \times 2\right] \qquad = \dfrac{2\mu_0 l}{\pi a}$

17.(D) $\therefore \qquad V_A - 0 = 4V \Rightarrow V_A = 4V$

According to question $V_B = 0$

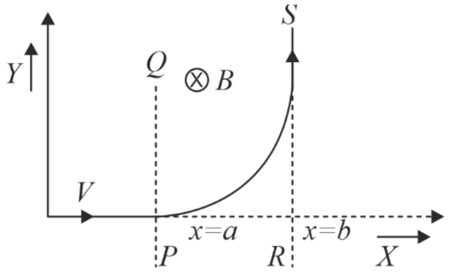
Point D is connected to positive terminal of battery of emf 3 V.

Using KVL between points B and D

$V_B - 6 + 3 = V_D$

$V_D = -3$ volts

18.(B) The particle moves in a circular path of radius r in the magnetic field. It can just enter the region $x > b$ for $r \geq (b-a)$.



Now, $\quad r = \dfrac{mv}{qB} \geq (b-a)$ Or $\quad v \geq \dfrac{q(b-a)B}{m} \Rightarrow v_{\min} = \dfrac{q(b-a)B}{m}$

19.(A) Due to increase in resistance R the current through the wire will decrease and hence the potential gradient also decreases, which results in increase in balancing length. So, J will shift towards B.

20.(B) $\because V = 3.2 \times 10^7 ms^{-1} \quad \Rightarrow \qquad B = 5 \times 10^{-4} T$

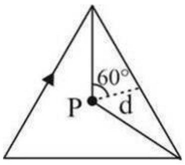
The frequency of electron, $\qquad v = \dfrac{qB}{2\pi m} = \dfrac{1.6 \times 10^{-19} \times 5 \times 10^{-4}}{2 \times 3.14 \times 9.1 \times 10^{-31}}$

$v = 1.4 \times 10^7 Hz = 14 MHz$

SECTION-2

1.(18) $B_p = 3\times\frac{\mu_0 i}{4\pi d}\left[\sin 60^\circ + \sin 60^\circ\right]$

$d = \frac{1}{2\sqrt{3}} \Rightarrow B_p = 18\times10^{-6}T = 18\mu T$



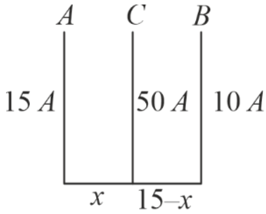
2.(12) When S_{W_1} is closed and S_{W_2} is open then capacitor B is charged upto 10 V so its find charge will be $3\times10 = 30\mu C$

Now S_{W_1} is open and S_{W_2} is closed then find common potential difference of the two capacitors in parallel be comes.

Thus find charges on capacitor are

$$V_C = \frac{30}{3+2} = 6V\ ;\ \ Q_A = 2\times10^{-6}V_{cm} = 12\mu C\ ;\ \ Q_B = 3\times10^{-6}V_{cm} = 18\mu C$$

3.(9)



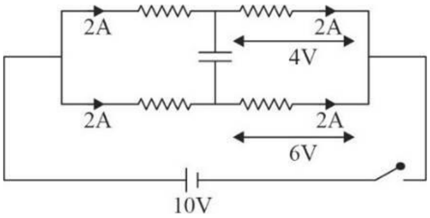
$$\frac{F_{AC}}{l} = \frac{F_{BC}}{l}$$

$$\frac{\mu_0}{4\pi}\cdot\frac{2\times15\times50}{x} = \frac{\mu_0}{4\pi}\cdot\frac{2\times50\times10}{15-x}$$

$$\frac{15}{x} = \frac{10}{(15-x)}$$

$225 - 15x = 10x \Rightarrow 25x = 255$ or $x = 9cm$

4.(6)



$Pd = 6-4 = 2V \qquad \Rightarrow \qquad Q = CV = 6\mu C$

5.(4) $R_{26} = \frac{2\times6}{2+6}\Omega = \frac{12}{8}\Omega$

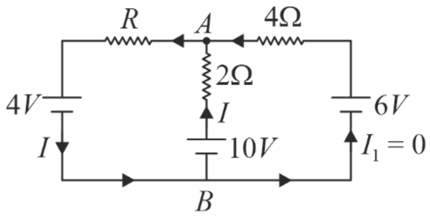
$R_{26} = \frac{3}{2}\Omega$

Is in series with 1.5Ω. This gives 3Ω. It is in parallel with 3Ω This $\frac{3}{2}\Omega$ *i.e.* $1.5\,\Omega$.

$$I = \frac{6}{1.5}A = 4A$$

6.(3) $Eq(2a-a)=\frac{1}{2}m(2v)^2-\frac{1}{2}mv^2 \Rightarrow E=\frac{3mv^2}{2aq}$

7.(1)



Since $I_1=0A, V_{AB}=6V$ ...(i)

and $I=\frac{(10-4)V}{(2+R)\Omega}=\frac{6}{2+R}A$

So, $V_{AB}=10-2I$

$\Rightarrow \quad V_{AB}=10-2\left(\frac{6}{2+R}\right)$ ...(ii)

Solving (i) and (ii) we get $R=1\Omega$

8.(48) $I=40\times10^{-6}\times30=1.2mA$

$\Rightarrow \quad B=\frac{4\pi\times10^{-7}\times1.2\times10^{-3}}{2\pi\ 5\times10^{-3}}=0.48\times10^{-7}T$

9.(10) $C=\frac{A\varepsilon_0}{\left[d+x_0-t\left(1-\frac{1}{k}\right)\right]}$

So, $x_0=t\left(1-\frac{1}{k}\right); \quad 4.5\times10^{-5}=5\times10^{-5}\left(1-\frac{1}{k}\right)$

$\Rightarrow \quad 1-\frac{1}{k}=0.9 \quad \Rightarrow \quad \frac{1}{k}=0.1\Rightarrow \quad k=10$

10.(4) $q=q_0e^{-t/\tau}$

At $t=2\sec$ $\Rightarrow$ $q_1=q_0e^{\frac{-2}{2/\ln 2}}=\frac{q_0}{2}$

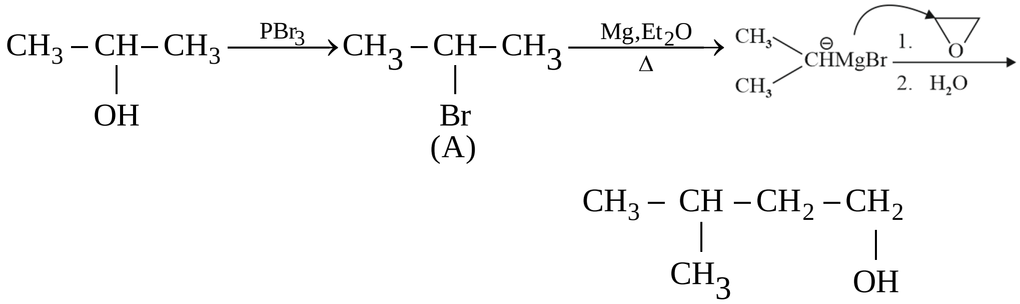
At $t=6\sec$ $\Rightarrow$ $q_2=q_0e^{\frac{-6}{2/\ln 2}}=\frac{q_0}{8}$

$\Rightarrow q_1:q_2::4:1$

CHEMISTRY

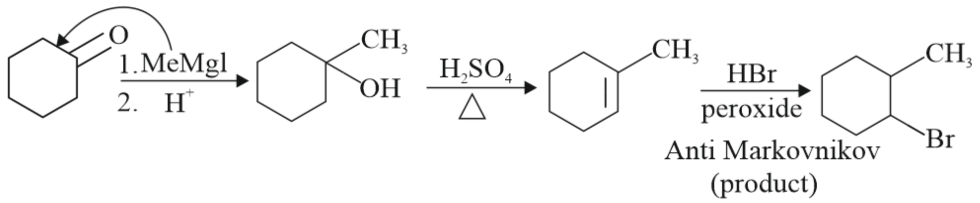
SECTION-1

1.(A)



2.(C) $CH_3-CH_2-\underset{\underset{O}{\|}}{C}-ONa \xrightarrow[\Delta]{\text{Sodalime}} CH_3-CH_3 \xrightarrow[\text{2. Na/dry ether}]{\text{1. } Br_2/h\nu} CH_3-CH_2-CH_2-CH_3$

3.(A)



4.(C) R-Br has higher boiling point than R-Cl
Boiling point $\propto$ molecular weight.

5.(B) **(II)** In Lead storage batteries, during charging cathode $\Rightarrow$ PbO_2
Anode = Pb
(III) In mercury cell, paste of KOH & ZnO acts as electrolytes.

6.(A) $\alpha = \frac{25}{125} = 0.2$; $Ka = \frac{c\alpha^2}{1-\alpha} = \frac{0.1 \times (0.2)^2}{1-0.2} = \frac{0.1 \times 0.04}{0.8} \quad = 5 \times 10^{-3}$

7.(A)

8.(C) **(II)** Enzymes are produced by living plants and Animals
(III) Enzymes form colloidal solution in water
(V) Enzymes are highly specific

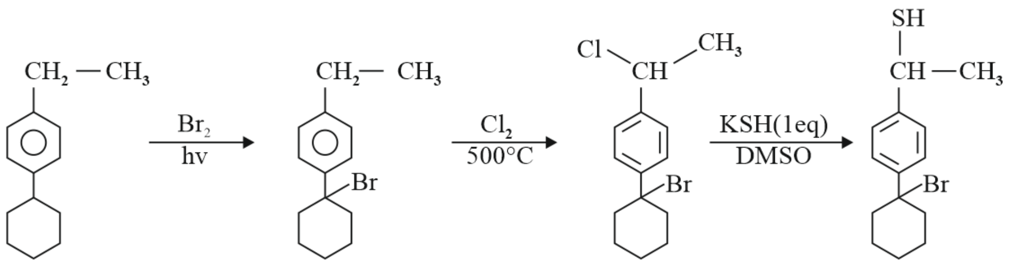
9.(B) Curves indicate that at a fixed pressure, there is a decrease in adsorption with increase in temperature.

10.(C) $2CHCl_3 + O_2 \xrightarrow{hv} \underset{\text{Phosgene}}{2COCl_2} + 2HCl$

11.(B)

12.(A) $\wedge_m^\circ(CH_3COOH) = \wedge_m^\circ(CH_3COOK) + \wedge_m^\circ(HBr) - \wedge_m^\circ(KBr)$
$= 142.7 + 423.7 - 117.6 = 448.8$

13.(A)



14.(C)

15.(B)

16.(A) $CH_2=CH-CH_3 \xrightarrow{Cl_2/500°C} CH_2=CH-CH_2Cl$

17.(D) Refer NCERT

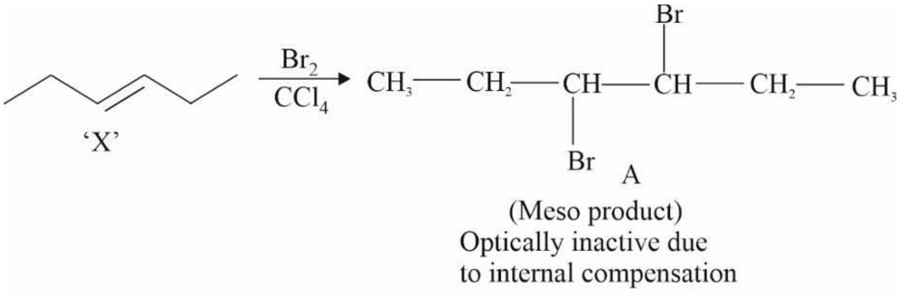
18.(C) Protective power is high if Gold number is low.

19.(B) $E_{cell}=1.5-\dfrac{0.059}{n}\log Q$; $E_{cell}=1.41V$

20.(B) Zn oxidises easily compared to Fe

SECTION-2

1.(1)



2.(11) $E_{cell}=E^o_{cell}-\dfrac{0.06}{2}\times\log\dfrac{[Zn^{2+}]}{[Fe^{2+}]}$

$0.3=E^o_{cell}-\dfrac{0.06}{2}\times\log\dfrac{0.1}{0.01}$

$0.33=E^o_{cell}$

At equilibrium

$\Delta G=0$ $\qquad$ $E^o_{cell}=0$

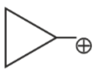
$Q=k_{eq}$

$0=0.33-0.03\log k_{eq}$; $\log k_{eq}=11$

3.(8) gm-eq of Aluminium produced = number of Faraday's of electricity passed

$\dfrac{27\times10^3}{9}=\dfrac{Q}{96500}$; $289500\times10^3=Q$; $x=8$

4.(4) Methylene blue sol, $Al_2O_3\cdot xH_2O, TiO_2$ and Haemoglobin are positive sols.

5.(6) Aromatic, Allylic, benzylic, 3° &  forms very stable carbocation.

6.(1) III is correct

7.(5) Amount of starch required to prevent coagulation on addition of 1 ml of 10% NaCl to 10 ml gold sol. $=0.005\,gm$ = 5 mg

Gold Number = 5

8.(3) $E^\circ_{cell}=1.23-0=1.23V$

$\Delta G^\circ_{cell}=-nFE^\circ_{cell}=-2\times96500\times1.23V=-237.4\,kJ/mol$

Work derived $=\dfrac{60}{100}\times\left(-\Delta G^\circ_{cell}\right)=\dfrac{60}{100}\times237.4\times10^3\times10^{-2}=1.425\times10^3\,J$; $x=3$

9.(36) Number of gm eq electrolysed = Number of Faraday's of charge passed

$=\dfrac{640}{106.5}\times6=36$

10.(2) (II), (III) are correct.

MATHEMATICS

SECTION-1

1. (B) (a) $$\text{R.H.D} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{h^{2/3}-0}{h} = \infty$$

$$\text{L.H.D} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}\frac{(-h)^{2/3}-0}{-h} = -\infty$$

$$\text{L.H.D} \neq \text{R.H.D}$$

So, vertical tangent is not exist.

(b) $$\text{R.H.D} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{Sgn(h)-0}{h} = \lim_{h\to 0}\frac{1}{h} = \infty$$

$$\text{L.H.D} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}\frac{Sgn(-h)-0}{-h} = \frac{-1}{-h} = \infty$$

R.H.D = L.H.D

So, vertical tangent exist.

(c) $$\text{R.H.D} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{\sqrt{|h|}}{h} = \infty$$

$$\text{L.H.D} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}\frac{\sqrt{|h|}}{-h} = -\infty$$

$$\text{R.H.D} \neq \text{L.H.D}$$

So, vertical tangent is not exist.

(d) $$\text{R.H.D} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{1-1}{h} = 0$$

$$\text{L.H.D} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}\frac{0-1}{-h} = \infty$$

$$\text{R.H.D} \neq \text{L.H.D}$$

So, vertical tangent is not exist.

2. (B) Let touch at (x_1, y_1)

So, $y_1 = e^{2x_1} = kx_1^2$

$y'_{(c_1)} = 2e^{2x_1}$ & $y'_{(c_2)} = 2kx_1$

So, $2e^{2x_1} = 2kx_1 \Rightarrow e^{2x_1} = kx_1$

$kx_1^2 = kx_1$

$x_1 = 0$ and $x_1 = 1$

So, $e^2 = k$

3. (A) $f'(x) = 5x^4 + 9x^2 + 4$

Clearly $f'(x) > 0$

So, $x \in R$

4. (B) $$\lim_{x\to 2}\frac{(x-2)\left[\sqrt{x-1}+\sqrt{3-x}\right]}{(x-1)-(3-x)} = \lim_{x\to 2}\frac{(x-2)\left[\sqrt{x-1}+\sqrt{3-x}\right]}{2(x-2)}$$

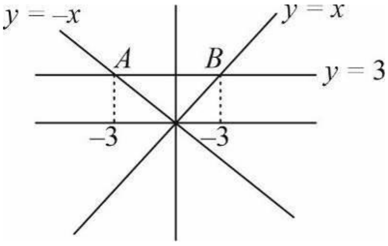
$$= \frac{\sqrt{2-1}+\sqrt{3-2}}{2} = 1$$

5. (B) $\lim_{x\to 0} \dfrac{e^{x^2}\left[e^{(\tan x^2 - x^2)} - 1\right]}{\tan x^2 - x^2} = 1\times 1 = 1$

6. (A) $\text{L.H.L} = \lim_{x\to 0^-} x^2 \cos e^{\frac{1}{x}} = 0$

$\text{R.H.L} = \lim_{x\to 0^+} \pi - \pi = 0$ So, $f(0) = 0$

7. (B)



At A & B graph has sharpness so at A & B function not differentiable.

8. (D)

$$\left.\begin{array}{l} f(0) = 0 \\ f\left(\dfrac{1}{4}\right) = 0 \end{array}\right\} \to \text{Critical pt} \in \left(0, \dfrac{1}{4}\right)$$

$$\left.\begin{array}{l} f\left(\dfrac{1}{9}\right) = 0 \end{array}\right\} \to \text{Critical pt} \in \left(\dfrac{1}{9}, \dfrac{1}{4}\right)$$

$$\left.\begin{array}{l} f\left(\dfrac{1}{16}\right) = 0 \end{array}\right\} \to \text{Critical pt} \in \left(\dfrac{1}{16}, \dfrac{1}{9}\right)$$

Infinite critical pt in $\left(0, \dfrac{1}{2}\right)$.

9. (B) $f'(x) = 12(x^3 - x^2 - x - 2) = 0$

$(x-2)(x^2 + x + 1) = 0$

$x = 2$ & $x^2 + x + 1$ always +ve

So, only one extremum point

10. (B) $f'(x) = -\sin 2x.2 - 2\cos x \sin x$

$f'(x) = -3\sin 2x$

For critical point

$f'(x) = 0$

$x = 0, \dfrac{\pi}{2}, \pi$

11. (B) $f'(x) = 20x + \dfrac{1}{x} = \dfrac{20x^2 + 1}{x} > 0$

$f'(x)$ is always +ve for $x > 0$ domain of $f(x)$ is R^+

So, for $x \in (0, \infty)$, curve increasing

12. (B) $f''(x) = 3x - 6$

$$f'(x) = \frac{3x^2}{2} - 6x + C_1$$

$$f'(1) = 3 \Rightarrow \frac{3}{2} - 6 + C_1 = 3 \Rightarrow C_1 = \frac{15}{2}$$

$$f'(x) = \frac{3x^2}{2} - 6x + \frac{15}{2} \;;\; f(x) = \frac{x^3}{2} - 3x^2 + \frac{15}{2}x + C_2$$

$$f(1) = 2 \Rightarrow \frac{1}{2} - 3 + \frac{15}{2} + C_2 = 2 \Rightarrow C_2 = -3 \qquad \text{So, } f(x) = \frac{x^3}{2} - 3x^2 + \frac{15}{2}x - 3$$

13. (D) $P'(x)$ extremum at $x = 4$ & $x = 5$

So, $P'(x) = K(x-4)(x-5) \Rightarrow P'(x) = K(x^2 - 9x + 20)$

$$P(x) = K\left(\frac{x^3}{3} - \frac{9x^2}{2} + 20x\right) + c$$

Put $P(0) = 1$ and $P(2) = 0$

So, $K = -\frac{3}{74}$ and $C = 1$

Hence, $P'(0) = 20K$

$$= 20 \times \left(\frac{-3}{74}\right)$$

$$P'(0) = -\frac{30}{37}$$

14. (B) $f'(x) = \frac{3ax^2}{3} + 2(a+2)x + 2(a-5)$

$$f''(x) = 2ax + 2(a+2) \Rightarrow x = \frac{-(a+2)}{a}$$

For +ve point of inflection, $x > 0$

$$\frac{-(a+2)}{a} > 0 \;;\; \frac{(a+2)}{a} < 0 \;;\; a \in (-2, 0)$$

15. (C) $\frac{dy}{dx} = 3e^{3x} + 6x$

at $x = 0$

$\frac{dy}{dx} = 3$ slope of tangent

Slope of normal $= \frac{-1}{3}$

at $x = 0,\ y = 1$

Equation of normal

$$y - 1 = -\frac{1}{3}(x - 0) \;;\; 3y - 3 = -x \;;\; x + 3y - 3 = 0$$

Perpendicular distance from origin to $x + 3y - 3 = 0$

$$= \frac{|0 + 0 - 3|}{\sqrt{1^2 + 3^2}} = \frac{3}{\sqrt{10}}$$

16. (D) Put $x = 0 = y$

$$f(0+0) = f(0) + \left(f(0)\right)^{2n+1} \Rightarrow f(0) = 0$$

Put $x = 0,\ y = 1$

$$f(1) = 0 + \left(f(1)\right)^{2n+1} \Rightarrow f(1) = 0, +1, -1 (f(1) \neq 0)$$

Differentiation w.r to y taken x as constant

$$f'(x + y^{2n+1})\left(2n+1\right)y^{2n} = (2n+1)(f(y))^{2n} f'(y)$$

Put y = 1

$$f'(x+1) = f'(1) \cdot (f(1))^{2n} = K(\text{constant})$$

Replace $x \to x-1$

$$\int g'(x) = \int k \Rightarrow f(x) = kx + k_1$$

Put $x = 0$ so, $k_1 = 0$

Hence $f(x) = kx$

If put $f(1) = -1$

So, $f(x) = -x$

$f'(x) = -1$

because $f'(x) \geq 0$

If put $f(1) = 1$

So, $f(x) = x$

$f'(x) = 1$

$f'(10) = 1$

17. (C) At $x = \sqrt{2}$, $\qquad f\left(\sqrt{2}\right) = 20$

So, $g'(y) = \dfrac{1}{f'(x)} \Rightarrow g'(20) = \dfrac{1}{f'\left(\sqrt{2}\right)}$

$$g'(x) = 8x + 12x^3 \Rightarrow f'\left(\sqrt{2}\right) = 32\sqrt{2}$$

So, $g^1(20) = \dfrac{1}{32\sqrt{2}}$

18. (B) Put $x = \cos\theta \Rightarrow \dfrac{\sqrt{3}}{2} = \cos\theta \Rightarrow \theta = \dfrac{\pi}{6}$

So, $f(\theta) = \tan^{-1}\left(\dfrac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}\right) \qquad \because \ \dfrac{\theta}{2} = \dfrac{\pi}{12}$

$f(\theta) = \tan^{-1}\tan\left(\dfrac{\pi}{4} - \dfrac{\theta}{2}\right) \qquad \therefore \ \dfrac{\pi}{4} - \dfrac{\theta}{2} = \dfrac{\pi}{4} - \dfrac{\pi}{12}$

$f(\theta) = \dfrac{\pi}{4} - \dfrac{\theta}{2} \qquad$ So, $\dfrac{-\pi}{2} < \dfrac{\pi}{4} - \dfrac{\theta}{4} < \dfrac{\pi}{2}$

$$f(x) = \frac{\pi}{4} - \frac{\cos^{-1}x}{2}\ ; \quad f'(x) = +\frac{1}{2}\frac{1}{\sqrt{1-x^2}}$$

$$f'\left(\frac{\sqrt{3}}{2}\right) = 1$$

19. (C) Let point $P(x_1, y_1)$ be closest to line $y = 2x - 5$

$$\left.\frac{dy}{dx}\right|_{(x_1,y_1)} = \text{Slope of line}$$

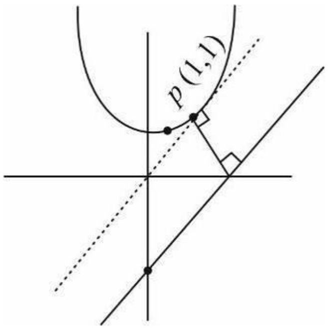
$6x_1 - 4 = 2$

$x_1 = 1$

So, $y_1 = 1$

$P(1,1)$

Perpendicular distance between P to line $= \left|\frac{2\times 1 - 1 - 5}{\sqrt{1+4}}\right| = \frac{4}{\sqrt{5}}$



20. (A) $f'(x) = 6x^{10} - 6x^5 + 6x^4 - 6x + 6$

$f'(x) = 6(x^{10} - x^5 + x^4 - x + 1)$

Case I: If $x < 0$

$$f'(x) = 6\left(\underset{+ve}{\underset{\downarrow}{x^{10}}} - \underset{+ve}{\underset{\downarrow}{x^5}} + \underset{+ve}{\underset{\downarrow}{x^4}} - \underset{+ve}{\underset{\downarrow}{x}} + 1\right) = +ve$$

Case II: If $x = 0$

$f'(x) = 1 = +ve$

Case III: If $0 < x < 1$

$$f'(x) = 6\left(\underset{+ve}{\underset{\downarrow}{x^{10}}} - \underbrace{x^5 + x^4}_{+ve} - \underbrace{x + 1}_{+ve}\right) = +ve$$

Case IV: If $x = 1$

$f'(x) = 1 = +ve$

Case V: If $x > 1$

$$f'(x) = 6\left(\underbrace{x^{10} - x^5}_{+ve} + \underbrace{x^4 - x}_{+ve} + 1\right) = +ve$$

So, $f(x)$ is increasing for $x \in R$

SECTION-2

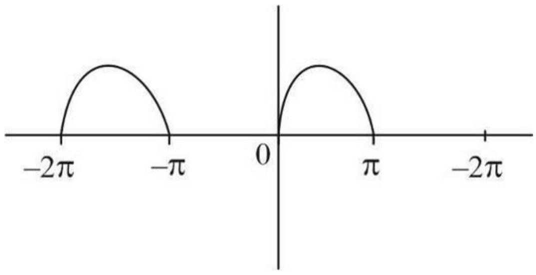
1.(4) $$\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{d}{dx}\left(\frac{1}{dy/dx}\right)\cdot\frac{dx}{dy}$$

$$=-\frac{1}{\left(\frac{dy}{dx}\right)^2}\cdot\frac{d^2y}{dx^2}\cdot\frac{1}{\frac{dy}{dx}}$$

$$\frac{d^2x}{dy^2}=-\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}\ ;\quad \frac{\left(\frac{d^2x}{dy^2}\right)\left(\frac{dy}{dx}\right)^3}{\left(\frac{d^2y}{dx^2}\right)}=-1$$

$$\frac{4\left(\frac{d^2x}{dy^2}\right)\left(\frac{dy}{dx}\right)^3}{\left(\frac{d^2y}{dx^2}\right)}=-4$$ So, $k=4$

2.(3) $$f(x)=\begin{cases}2\sin x, & \sin x\ge 0\\ 0, & \sin x<0\end{cases}$$



Sharpness in graph at $-\pi, 0, \pi$

So, Number of non-differentiable point = 3

3.(3) $g'(x)=f'(\sin x).\cos x-f'(\cos x).\sin x\ge 0$

$f'(\sin x)\cdot\cos x\ge f'(\cos x)\cdot\sin x$

Let $h(x)=f'(\sin x).\cos x$

So, $h(x)\ge h\left(\frac{\pi}{2}-x\right)$

$h'(x)=f''(\sin x)\cos x.\cos x-f'(\sin x)\cdot\sin x$

$h'(x)=+ve$

So, $x\ge\frac{\pi}{2}-x$

$x\ge\frac{\pi}{4}$

Hence, $g(x)$ increasing in $\left[\frac{\pi}{4},\pi\right)$

So, Number of integral point = 3

4.(5) For continuous at $x = 0$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0) = 0$$

$\therefore$ m must be $\Rightarrow$ $m > 0$

$$f'(0^+) = f'(0^-) = \lim_{x\to 0} \frac{x^{\frac{m}{5}} \sin\left(\frac{1}{x}\right) - 0}{x}$$

$$= \lim_{x\to 0} x^{\frac{m}{5}-1} \sin\frac{1}{x}$$

For non-differentiable $\frac{m}{5} - 1 \le 0 \Rightarrow m \le 5$

So, $m = 5$

5.(3) $f(x) = 2x^3 - 6x + 3\lambda$

$f'(x) = 6x^2 - 6 = 0$

$x = \pm 1$

For three real and distinct roots $f(1)\cdot f(-1) < 0$

$(2 - 6 + 3\lambda)\ (-2 + 6 + 3\lambda) < 0 \Rightarrow (3\lambda - 4)(3\lambda + 4) < 0$

$9\lambda^2 - 16 < 0 \quad \Rightarrow \lambda \in \left(-\frac{4}{3}, \frac{4}{3}\right)$

Integral value of $\lambda = -1, 0, 1$

No. of integral value of $\lambda = 3$

6.(2) Put $x = \frac{\pi}{3} + h$

$$\lim_{h\to 0} \frac{\sin h}{\cos\frac{\pi}{3} - \cos\left(\frac{\pi}{3} + h\right)} = \lim_{h\to 0} \frac{\sin h}{2\sin\left(\frac{\pi}{3} + \frac{h}{2}\right)\cdot \sin\left(\frac{h}{2}\right)}$$

$$\lambda = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$\sqrt{3}\lambda = 2$

7.(3) $x^3 = 12y$

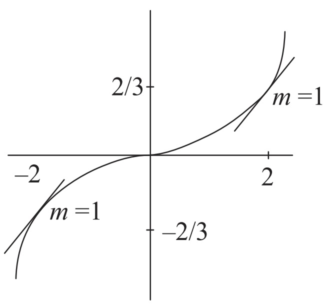
$3x^2 = 12\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x^2}{4}$

$\frac{x^2}{4} < 1 \Rightarrow x^2 < 4$

$\Rightarrow x \in (-2, 2)$

$\Rightarrow x = -1, 0, 1$

Number of integral value = 3



8.(38) $\frac{dy}{dx}=\frac{x^2}{2}-6x \Rightarrow y=\frac{x^3}{6}-3x^2+c$

Put (2,2)

$2=\frac{8}{6}-3\times4+C \Rightarrow C=14-\frac{8}{6} \Rightarrow C=\frac{38}{3}$

$y=\frac{x^3}{6}-3x^2+\frac{38}{3}$ ...(1)

$\frac{dy}{dx}=0 \Rightarrow \frac{x^2}{2}-6x=0 \Rightarrow x=0,12$

$\frac{d^2y}{dx^2}=x-6$

at $x=0, \frac{d^2y}{dx^2}=-ve$

at $x=12, \frac{d^2y}{dx^2}=+ve$

So y max at $x = 0$

$y=0-0+\frac{38}{3}$

$y=\frac{38}{3}$

So, $3\lambda=38$

9.(1) $\lim_{x\to\infty}\left(x^2-\frac{x}{2}\right)-x^3\left[\frac{1}{x}-\frac{1}{2x^2}+\frac{1}{3x^3}-\frac{1}{4x^4}+......\right]$

$=\lim_{x\to\infty} x^2-\frac{x}{2}-x^2+\frac{x}{2}-\frac{1}{3}+\frac{1}{4x}---$

$\lambda=-\frac{1}{3}$

$|3\lambda|=1$

10.(2) $\lim_{x\to0}\frac{x\left[3x+\frac{(3x)^3}{3}+--\right]-3x\left[x+\frac{x^3}{3}+--\right]}{\left[1-(1-2\sin^2 x)\right]^2}=A$

$A=\lim_{x\to0}\frac{8x^4+...}{4\sin^4 x}=2$

$A=2$